ROBUST POLE PLACEMENT

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Abstract: This Dissertation is dedicated to Robust Pole Placement. In Robust Pole Placement Controller, It has shown easiest in terms of control the State Space Design method and a wide range of problem solving abilities of Pole Placing. The Robust Pole Placement assignment algorithm. I explain an unconstrained nonlinear optimization method to obtain a gain matrix that delivers robust pole placement or minimum gain. This topology is used to derive equations for the state feedback gain matrix K for the pole placement in design a control scheme. The problem of computing a well-conditioned solution to this equation has been addressed, reducing it to the minimization of the condition number. The Simulation will results of MATLAB/SIMULINK model indicate the performance of the proposed control system as well as the precision of the proposed topology.
Key Words: Inverted Pendulum, Robust Pole placement, Mathematical and computational calculation

I. INTRODUCTION

Nowadays, state –space design methods based on the pole placement method and the quadratic optimal regulator method. The pole placement is somewhat similar to the root locus method in that we placed close-loop poles at desire location. The basic difference is that is the root locus, In root locus design we placed only the dominant closed loop poles at the desired locations, while in the pole placement design we placed all close loop poles at desired location. We begin by presenting the basic materials on the poles placement in regulator systems. We then discuss the design of state observers, and after by the design of regulators systems and control systems using the pole placement –with –state –observer approach. Finally we present the quadratic optimal regulator system Full state feedback (FSF), or pole placement, is a method employed in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the s-plan. Placing poles is desirable because the location of the poles corresponds directly to the eigenvalues of the system, which control the characteristics of the response of the system. The necessary and sufficient condition that the closed loop poles can be placed at any desired location through state feedback by means of an appropriate state feedback gain matrix.

II. ROBUST POLE PLACEMENT

Pole assignment technique is the method for designing of control system. The Robust Pole Placement technique describes all the closed loop poles which require measurements of all state variables, function or inclusion of a state observer in the system. The condition for taken system is completely controllable for all close loop poles which can be placed at arbitrary chosen location.In this chapter we discuss state-space design methods based on the pole-placement method, Observers, the quadratic optimal regulator systems, and introductory aspects of robust control systems. The pole-placement method is somewhat similar to the root-locus method in that we place closed-loop poles at desired locations. The basic difference is that in the root-locus design we place only the dominant closed-loop poles at the desired locations, while in the pole-placement design we place all closed-loop poles at desired locations. We begin by presenting the basic materials on pole placement in regulator systems. We then discuss the design of state observers, followed by the design of regulator systems and control systems using the pole-placement with-state-observer approach. Then we discuss the quadratic optimal regulator systems. And in the last we present an introduction to robust control systems. A modern control or the complex system may have multiple inputs and multiple outputs, and these may be consistent in a complicated manner. To study such a system, it is essential to draw out the complexity of the mathematical expressions, as well as to resort to computers for most of the tedious computations necessary in the analysis. The state-space approach to system analysis is best suited from this viewpoint. While conventional control principle is based on the input–output relationship, or transfer function, modern control theory is based on the description of system equations in terms of n first-order differential equations, which may be joined into a first-order vector-matrix differential equation. The use of vector-matrix notation easily simplifies the mathematical calculation or representation of systems of equations. The increase in the number of state variables or the number of inputs or the number of outputs does not increase the complexity of the equations. In fact, the research of complicated multiple-input, multiple output systems can be carried out by procedures that are only slightly more complicated than those required for the analysis of systems of first-order scalar differential equations.

A. Pole Placement

We shall present a design method commonly called the pole-placement or pole-assignment technique. We assume that all state variables are measurable and are available for feedback. It will be shown that if the system considered is completely state controllable, then poles of the closed-loop system may be placed at any desired locations by means of state feedback through an appropriate state feedback gain matrix. The present design technique begins with a determination of the desired closed-loop poles based on the transient-response and/or frequency-response requirements, such as speed, damping ratio, or bandwidth, as well as steady-state requirements. Let us assume that we decide that the desired closed-loop poles are to be at s=1, s=2…s=n . By choosing an appropriate gain matrix for state feedback, it is possible to force the system to have closed-loop poles at the desired locations, provided that the original system is completely state controllable. N In this chapter we limit our discussions to single-input, single-output systems. That is, we assume the control signal u (t) and output signal y (t) to be scalars. In the derivation in this section we assume that the reference input r (t) is zero. In what follows we shall prove that a necessary and sufficient condition that the closed-loop poles can be placed at any arbitrary locations in the s plane is that the system be completely state controllable. Then we shall discuss methods for determining the required state feedback gain matrix. It is noted that when the control signal is a vector quantity, the mathematical aspects of the pole-placement scheme become complicated. We shall not discuss such a case in this book. (When the control signal is a vector quantity, the state feedback gain matrix is not unique. It is possible to choose freely more than n parameters; that is, in addition to being able to place n closed-loop poles properly, we have the freedom to satisfy some or all of the other requirements, if any, of the closed-loop system.)

B. Design by Pole Placement

In the conventional approach to the design of a single input single-output control system, we design a controller (compensator) such that the dominant closed-loop poles have a desired damping ratio and a desired undamped natural frequency. In this approach, the order of the system may be raised by 1 or 2 unless pole–zero cancellation takes place. Note that in this approach we assume the effects on the responses of non-dominant closed-loop poles to be negligible.

Different from specifying only dominant closed-loop poles (the conventional design approach), the present pole-placement approach specifies all closed-loop poles. (There is a cost associated with lacing all closed-loop poles, however, because placing all closed-loop poles requires successful measurements of all state variables or else requires the inclusion of a state observer in the system.) There is also a requirement on the part of the system for the closed-loop poles to be placed at arbitrarily chosen locations. The requirement is that the system be completely state controllable. We shall prove this fact in this section.

Consider a control system,

$$Xdot(t) = AX(t) + BU(t),$$
$$Y(t) = CX(t) + DU(t). \qquad \textbf{(3-1)}$$

Where,

X dot (t) = State vector matrix of order n x 1.

U (t) = Input vector matrix of order m x 1.

A = System Matrix or Evolution Matrix of order n x n.
B = Input matrix or control matrix n x m.
Y (t) = Output vector matrix of order p x 1.
C = Output matrix or observation matrix of order p x m.
D = Direct transmission matrix of order p x m.
We shall choose the control signal to be
U = - K x. **(3-2)**

Where X (0) is the initial state caused by external disturbances. The stability and transient response characteristics are determined by the eigenvalues of matrix **A-BK.** If matrix K is chosen properly, the matrix A-BK can be made an asymptotically stable matrix, and for all x (0) Z 0, it is possible to make x(t) approach 0 as t approaches infinity. The eigenvalues of matrix A-BK are called the regulator poles. If these regulator poles are placed in the left-half s plane, then x(t) approaches 0 as t approaches infinity. The problem of placing the regulator poles (closed-loop poles) at the desired location is called a pole-placement problem. In what follows, we shall prove that arbitrary pole placement for a given system is Possible if and only if the system is completely state controllable. This means that the control signal U is determined by an instantaneous state. Such a scheme is called state feedback. The 1 x n matrix **K** is called the state feedback gain matrix. We assume that all state variables are available for feedback. In the following analysis we assume that u is unconstrained. A block diagram for this system is shown in Figure 3.1

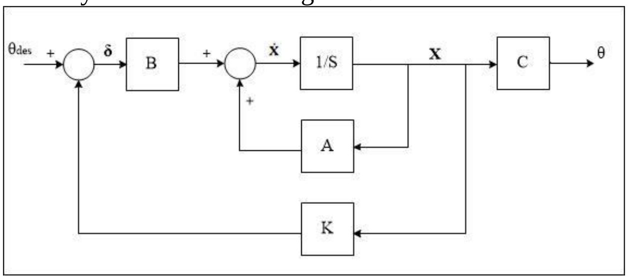

Figure 1 Full-state feedback control system (whereD=0)

This closed-loop system has no input. Its objective is to maintain the zero output. Because of the disturbances that may be present, the output will deviate from zero. The nonzero output will be returned to the zero reference input because of the state feedback scheme of the system.

Such a system where the reference input is always zero is called a regulator system. (Note that if the reference input to the system is always a nonzero constant, the system is also called a regulator system.) Substituting Equation (3–2) into Equation (3–1) gives

X dot(t) = (A - BK) x(t).

The solution of this equation is given by

$$X(t) = e^{(A-BK)t}x(0) \qquad \textbf{(3-3)}$$

Where x (0) is the initial state caused by external disturbances. The stability and transient response characteristics are determined by the eigenvalues of matrix A-BK. If matrix K is chosen properly, the matrix A-BK can be made an asymptotically stable matrix, and for all x(0) Z 0, it is possible to make x(t) approach 0 as t approaches infinity. The eigenvalues of matrix A-BK are called the regulator poles. If these regulator poles are placed in the left-half s plane, then x(t) approaches 0 as tapproaches infinity. The problem of placing the regulator poles (closed-loop poles) at the desired location is called apole-placement problem. In what follows, we shall prove that arbitrary pole placement for a given system is possible if and only if the system is completely state controllable.

Applications

- State-Space Methods for Controller Design.
- Cruise Control.
- Aircraft Pitch.
- Inverted Pendulum.
- Ball and Beam.

III. SIMULATION WORK

A. Inverted Pendulum

```
clc
clearall
%****The following program is to obtain step response
%of the Inverted -pendulum system just designed*****
A=[0 1 0 0;20.601 0 0 0;0 0 0 1;-0.4905 0 0 0];
B=[0;-1;0;0.5];
C=[0 0 1 0];
D=[0];
K=[-157.6336 -35.3733 -56.0652 -36.7466];
KI=-50.9684;
AA=[A-B*K B*KI;-C 0];
BB=[0;0;0;0;1];
CC=[C 0];
DD=[0];

%*****To obtain curvesx1 versus t,x2 versus t,
%x3 versus t,x4 versus t,x5 versus t, seprately,enter
%the following command*****

t=0:0.02:6;
[y,x,t]=step(AA,BB,CC,DD,1,t);

x1=[1 0 0 0 0]*x';
x2=[0 1 0 0 0]*x';
x3=[0 0 1 0 0]*x';
x4=[0 0 0 1 0]*x';
x5=[0 0 0 0 1]*x';

subplot(3,2,1);plot(t,x1);grid
title('x1 versus t');
xlabel('t Sec');
ylabel('x1');

subplot(3,2,2);plot(t,x2);grid
title('x2 versus t');
xlabel('t Sec');
ylabel('x2');

subplot(3,2,3);plot(t,x3);grid
title('x3 versus t');
xlabel('t Sec');
ylabel('x3');

subplot(3,2,4);plot(t,x4);grid
title('x4 versus t');
xlabel('t Sec');
ylabel('x4');

subplot(3,2,5);plot(t,x5);grid
title ('x5 versus t');
xlabel('t Sec');
ylabel('x5');
```

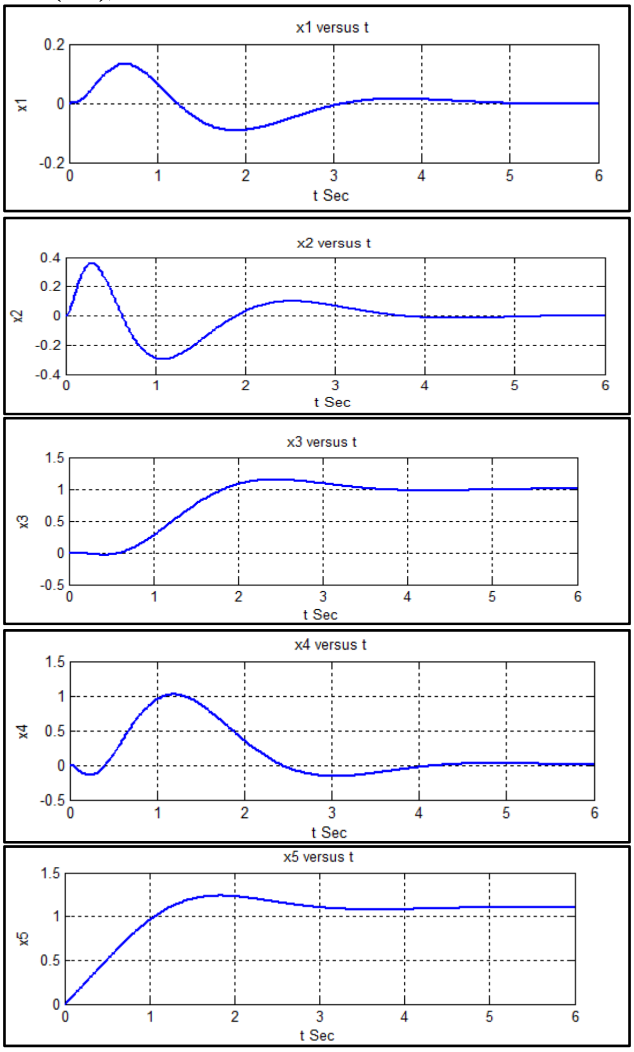

Fig. Output result of inverted pendulum

```
clc
clearall
%**** Consider the Third order continueus-time system
%treated in x dot= Ax(t)+Bu(t)*****

A=[0 1 2;-2 3 0;-2 -1 0];
B=[1 2;1 0;0 0];

%**** Let the desired eigenvalues as below

s1=-1;
m1=2;
```



```
q1=1;
s2=-2;
m2=1;
q2=1;
p11=2;
p21=2;

for r=-20:1:20
f110=[0;1];
f111=[r/10;0];
f210=[1;1];
v=-eye(3)
v110=inv(v-A)*B*f110
v111=-inv(v-A)^2*B*f110+inv(v-A)*B*f111
v210=inv(-2*eye(3)-A)*B*f210
v=[v110,v111,v210]
K=[f110,f111,f210]*inv(v)
Ac=A+(B*K)
J=inv(v)*Ac*v
norm_K(r+21)=sqrt(trace(K*K'))
end
r=-20:1:20
plot(r/10,norm_K(r+21))
title('K versus r');
xlabel('r');
ylabel('K');
```

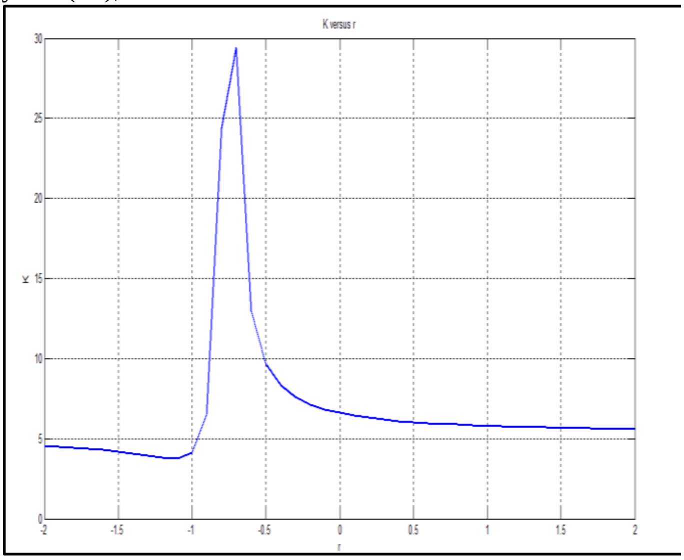

Fig. Close loop Eigen structures via state feedback to a linear multivariable system

IV. CONCLUSION

Robust Pole Placement has most promising technique for Design of control systems in state space application. The state-space design method based on the pole-placement-combined-with observer approach is very powerful. It is a time-domain method. The desired closed loop poles can be arbitrarily placed, provided the plant is completely state controllable. This parametric form was used to formulate the robust and minimum gain exact pole placement problems as constrained optimization problems, to be solved by gradient iterative methods.

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