

DIGITAL WATER MARKING TECHNIQUES: AN EVALUATION

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Abstract: *The sudden increase in watermarking interest is most likely due to the increase in concern over copyright protection of content. With the rapid growth of the Internet and the multimedia systems in distributed environments, digital data owners are now easier to transfer multimedia documents across the Internet. However, current technology does not protect their copyrights properly. This leads to wide interest of multimedia security and multimedia copyright protection and it has become a great concern to the public in recent years. In the early days, encryption and control access techniques were used to protect the ownership of media. Recently, the watermark techniques are utilized to keep the copyrights. This paper will discuss what requirements a watermarking system must meet. The remainder of the paper will focus on various watermarking techniques. Digital image watermarking is one such technology that has been developed to protect digital images from illegal manipulations. In particular, digital image watermarking algorithms which are based on the discrete wavelet transform have been widely recognized to be more prevalent than others. This is due to the wavelets' excellent spatial localization, frequency spread, and multi-resolution characteristics, which are similar to the theoretical models of the human visual system. In this paper, we describe an imperceptible and a robust combined DWT-DCT digital image watermarking algorithm. The algorithm watermarks a given digital image using a combination of the Discrete Wavelet Transform (DWT) and the Discrete Cosine Transform (DCT). Performance evaluation results show that combining the two transforms improved the performance of the watermarking algorithms that are based solely on the DWT transform. The third part of the paper describes the construction of arrays suitable for embedding as watermarks. We have also explained the contrary requirements of a good watermark and contrast them to the features of spread-spectrum communications.*

Index Terms – Digital Image Processing, Digital Water Marking, DCT, DWT and CDMA

I. INTRODUCTION

The earliest forms of information hiding can actually be considered to be highly crude forms of private-key cryptography; the “key” in this case being the knowledge of the method being employed (security through obscurity). Steganography books are filled with examples of such methods used throughout history. Greek messengers had messages tattooed into their shave head, concealing the message when their hair finally grew back. Wax tables were scraped down to bare wood were a message was scratched.

Once the tablets were re-waxed, the hidden message was secure. Over time these primitive cryptographic techniques improved, increasing both speed, capacity and security of the transmitted message. Today, crypto-graphical techniques have reached a level of sophistication such that properly encrypted communications can be assumed secure well beyond the useful life of the information transmitted. In fact, it's projected that the most powerful algorithms using multi kilobit key lengths could not be comprised through brute force, even if all the computing power worldwide for the next 20 years was focused on the attack. Of course the possibility exists that vulnerabilities could be found, or computing power breakthroughs could occur, but for most users in most applications, current cryptographic techniques are generally sufficient. Several good reasons exist, the first being that “security through obscurity” isn't necessarily a bad thing, provided that it isn't the only security mechanism employed. Steganography for instance allows us to hide encrypted messages in mediums less likely to attract attention. A garble of random characters being transmitted between two users may tip off a watchful 3rd party that sensitive information is being transmitted; whereas baby pictures with some additional noise present may not. The underlying information in the pictures is still encrypted, but attracts far less attention being distributed in the picture than it would otherwise. This becomes particularly important as the technological disparity between individuals and organizations grows. Governments and businesses typically have access to more powerful systems and better encryption algorithms than individuals. Hence, the chance of individual's messages being broken increases which each passing year. Reducing the number of messages intercepted by the organizations as suspect will certainly help to improve privacy. Another advantage hinted at by A. Tewfik is that information hiding can fundamentally change the way that we think about information security. Cryptographic techniques generally rely on the metaphor of a piece of information being placed in a secure “box” and locked with a “key”. The information itself is not disturbed and anyone with the proper key can gain access. Once the box is open, all of the information security is lost. Compare this to information hiding techniques where the key is embedded into the information itself. This difference can be better illustrated by current DVD encryption methods. The CSS algorithm takes digitally encoded video and wraps it in an encrypted container. When the DVD player provides the proper key, the video is decrypted and played. Once the video has been decrypted even once, it becomes trivial to trans-code the content and distributes it with no mark of the

original author present. Compare this approach to that of an ideal watermark, where despite encryption the watermark remains with the video despite various alteration and trans-coding attempts. With this the need for a combination of the two approaches becomes clear. This paper will begin with a quick background on cryptography and steganography, which form the basis for a large number of digital watermarking concepts. This paper will then move on to a discussion of what requirements a watermarking system must meet, as well as methods for evaluating the strengths of various algorithms. The remainder of the paper will focus on various watermarking techniques and the strengths and weaknesses of each. This paper will focus almost exclusively on the watermarking of digital images, however most of these same ideas could easily be applied to the watermarking of digital video and audio.

II. DIGITAL WATER MARKING

This technique describes the different approaches that watermarking algorithms is based on. The main topics being whether to embed watermarks in the spatial domain or a transform domain. Advantages and disadvantages are also discussed.

A. The Ideal Algorithm

Nothing is perfect, which also holds true for watermarking algorithms. But what if it was possible to construct the perfect or ideal watermarking scheme? Listed here are sums up of requirements to such an ideal algorithm, serving as introduction to the topic, but also emphasizing the many contradictions one encounters.

B. Robust

Since watermarking is primarily used for copyright protection and proving ownership, the embedded watermark has to survive and be extractable after the marked image has been submitted to a variety of things, for example:

- Scaling of the image
- Converting a color image to grayscale
- Blurring, sharpening and other image-effect algorithms
- Lossy compression, for example JPEG, widely used on the internet

C. Transparent

There are some obvious reasons for wanting to embed the watermark, without being able to see any difference on the marked image contra the original. Not being able to see the watermark, may keep some people from trying to remove it. If the image is used unrightfully, and your watermark can afterwards be extracted, you have a pretty good case against the copyright violator. It is also desirable to preserve the quality of an image, even though a watermark is embedded in it. Imagine for example that beautiful pictures promoting a tourist website are severely distorted by the watermarking. Then the algorithm would be practically unusable.

D. Tamper Resistant

Tightly linked to robustness, since any effort made to remove or deteriorate the watermark should result in the watermarked image being severely degraded in quality. There are different approaches for achieving a good level of robustness, which will be discussed later.

E. Cheap and Easy Implementation

For a watermarking algorithm to have success, it has to be relatively easy to implement, while not costing a fortune. An algorithm is of no use if it takes a day to mark a picture, and a day to extract the mark again. It has to be usable in real life which of course is application dependant

F. Robust and Fragile Water marks

It seems that for most applications, it would be ideal to have watermarks that are able to survive transmission, usage and attacks. Such a watermark is named robust. On the other hand, a watermark is also used to detect if the image they are in, has been altered. That is watermarks that cannot resist any alteration. Such watermark is called fragile. Finally watermarks have been proposed, trying to combine robustness and fragility. That is a watermark that can survive some alterations, but would break if the image was cropped for example, or a part of another image was inserted into it

G. Spatial Domain

One approach is to embed watermarks in the spatial domain of an image as shown in Figure 1. The idea is that the number of bits you wish to embed, the watermark size, is stored in the pixels of the image to be marked. Let us for example want to embed 40 bits of watermark information in a host image. Then we supply a seed to a PRNG (Pseudo Random Number Generator) and uses it to provide us with 40 sets of (x, y) with respect to the illustration, in other words we select 40 pixels.

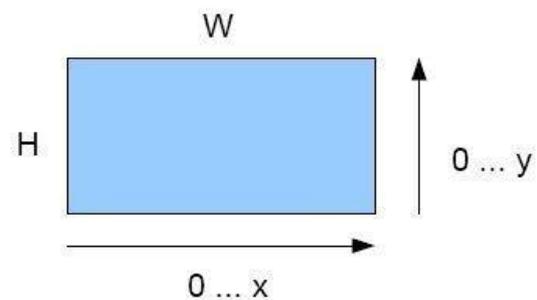


Fig.1 A Simple Digital Image Representation

Each of these pixels are now altered, exchanging a chosen bit of color information with our watermark bit. The choice of bit involves a trade-off:

If we choose one of the lesser significant bits, our alteration can be held invisible in the marked picture, but image compression and other things might be attacking these less significant bits, which could mean that the watermark is not very robust.

If a more significant bit is chosen, we would achieve much better robustness, but our alteration would be visible. It

would look weird if the sunset picture had 40 black spots in it. When you would attempt to extract the watermark at a given time, the PRNG would then be started with the same seed, and again produce the 40 pixels, from where we can extract our bits.

This method is easy to use, but has many drawbacks. Even small alterations to our image would destroy at least some of our watermark information. Therefore the method could be used for embedding fragile watermarks

III. DCT & DWT TRANSFORMATIONS

The DCT equation computes i, jth entry of the DCT of an image,

$$D(i,j) = \frac{1}{\sqrt{2N}} C(i)C(j) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} P(x,y) \cos \left[\frac{(2x+1)i\pi}{2N} \right] \cos \left[\frac{(2y+1)j\pi}{2N} \right]$$

$$C(u) = \frac{1}{\sqrt{2}} \begin{cases} 1 & \text{if } u=0 \\ \sqrt{2} & \text{if } u>0 \end{cases}$$

P(x,y) is the x&y element of the image represented by the matrix p. N is the size of the block that the DCT is done on. The equation calculates one entry (ith) of the transformed image from the pixel values of the original image matrix. For the standard 8x8 block that JPEG compression uses, N equals 8 and x and Y range from 0 to 7.

The equation D(i,j) is given by

$$D(i,j) = C(i)C(j) \sum_{x=0}^7 \sum_{y=0}^7 P(x,y) \cos \left[\frac{(2x+1)i\pi}{2N} \right] \cos \left[\frac{(2y+1)j\pi}{2N} \right]$$

Because the DCT uses cosine functions, the resulting matrix depends on the horizontal, diagonal, and vertical frequencies. Therefore an image black with a lot of change in frequency has a very random looking resulting matrix, while an image matrix of just one color, has a resulting matrix of a large value for the first element and zeroes for the other elements.

A. DCT Matrix

To get the matrix form of Equation

$$T_{ij} = \begin{cases} \frac{1}{\sqrt{N}} & ; \text{if } i=0 \\ \sqrt{\frac{2}{N}} \cos \left[\frac{(2y+1)j\pi}{2N} \right] & ; \text{if } i>0 \end{cases}$$

For an 8x8 block it results in this matrix:

$$T = \begin{bmatrix} .3536 & .3536 & .3536 & .3536 & .3536 & .3536 & .3536 & .3536 \\ .4904 & .4157 & .2778 & .0975 & -.0975 & -.2778 & -.4157 & -.4904 \\ .4619 & .1913 & -.1913 & -.4619 & -.4619 & -.1913 & .1913 & .4619 \\ .4157 & -.0975 & -.4904 & -.2778 & .2778 & .4904 & .0975 & -.4157 \\ .3536 & -.3536 & -.3536 & .3536 & .3536 & -.3536 & -.3536 & .3536 \\ .2778 & -.4904 & .0975 & .4157 & -.4157 & -.0975 & .4904 & -.2778 \\ .1913 & -.4619 & .4619 & -.1913 & -.1913 & .4619 & -.4619 & .1913 \\ .0975 & -.2778 & .4157 & -.4904 & .4904 & -.4157 & .2778 & -.0975 \end{bmatrix}$$

The first row(i=0) matrix of all entry is equal to $\frac{1}{\sqrt{8}}$ as expected. If we want inverse DCT of T we can be easily obtained as T.

B. Doing the DCT on an 8x8 Block

Before we begin, it should be noted that the pixel values of a black-and-white image range from 0 to 255 in steps of 1, where pure black is represented by 0, and pure white by 255. Thus it can be seen how a photo, illustration, etc. can be accurately represented by these 256 shades of gray. Since an image comprises hundreds or even thousands of 8x8 blocks of pixels, the following description of what happens to one 8x8 block is a microcosm of the JPEG process. Now, let's start with a block of image-pixel. This particular block was chosen from the very upper left hand corner of an image, because the DCT is designed to work on pixel values ranging from -128 to 127. The original block is levelled off by subtracting 128 from each entry. This results in the following matrix.

$$Original = \begin{bmatrix} 154 & 123 & 123 & 123 & 123 & 123 & 123 & 136 \\ 192 & 180 & 136 & 154 & 154 & 154 & 136 & 110 \\ 254 & 198 & 154 & 154 & 180 & 154 & 123 & 123 \\ 239 & 180 & 136 & 180 & 180 & 166 & 123 & 123 \\ 180 & 154 & 136 & 167 & 166 & 149 & 136 & 136 \\ 128 & 136 & 123 & 136 & 154 & 180 & 198 & 154 \\ 123 & 105 & 110 & 149 & 136 & 136 & 180 & 166 \\ 110 & 136 & 123 & 123 & 123 & 136 & 154 & 136 \end{bmatrix}$$

$$M = \begin{bmatrix} 26 & -5 & -5 & -5 & -5 & -5 & -5 & 8 \\ 64 & 52 & 8 & 26 & 26 & 26 & 8 & -18 \\ 126 & 70 & 26 & 26 & 52 & 26 & -5 & -5 \\ 111 & 52 & 8 & 52 & 52 & 38 & -5 & -5 \\ 52 & 26 & 8 & 39 & 38 & 21 & 8 & 8 \\ 0 & 8 & -5 & 8 & 26 & 52 & 70 & 26 \\ -5 & -23 & -18 & 21 & 8 & 8 & 52 & 38 \\ -18 & 8 & -5 & -5 & -5 & 8 & 26 & 8 \end{bmatrix}$$

We are now ready to perform the discrete Cosine Transform. This is matrix multiplication. D=TMT^TMatrix M is first multiplied on the left by the DCT matrix T from the previous section this transforms the rows. The columns are then transformed by multiplying on the right by the transpose of the DCT matrix. This yields the following matrix.

$$D = \begin{bmatrix} 162.3 & 40.6 & 20.0 & 72.3 & 30.3 & 12.5 & -19.7 & -11.5 \\ 30.5 & 108.4 & 10.5 & 32.3 & 27.7 & -15.5 & 18.4 & -2.0 \\ -94.1 & -60.1 & 12.3 & -43.4 & -31.3 & 6.1 & -3.3 & 7.1 \\ -38.6 & -83.4 & -5.4 & -22.2 & -13.5 & 15.5 & -1.3 & 3.5 \\ -31.3 & 17.9 & -5.5 & -12.4 & 14.3 & -6.0 & 11.5 & -6.0 \\ -0.9 & -11.8 & 12.8 & 0.2 & 28.1 & 12.6 & 8.4 & 2.9 \\ 4.6 & -2.4 & 12.2 & 6.6 & -18.7 & -12.8 & 7.7 & 12.0 \\ -10.0 & 11.2 & 7.8 & -16.3 & 21.5 & 0.0 & 5.9 & 10.7 \end{bmatrix}$$

This block matrix consist of 64 DCT co-efficient C_{ij} , where, i & j are the range from 0 to 7. The top left coefficient, C_{00} , correlates to the low frequency of the original image. As we move away from C_{00} in all direction, the DCT coefficients correlates to higher and higher frequency of the image block.

C. DWT Matrix

A wavelet transform can be interpreted as decomposition into a set of frequency channels having the same bandwidth on a logarithmic scale. The basic idea of the discrete wavelet transform is that of successive approximation, together with that of “added detail”. At each stage, the input signal is decomposed into a coarse approximation signal (which can be considered as a low pass version of the input) and an “added detail” signal (which can be considered as a high pass version). One-dimensional discrete wavelet transforms (the separable 2-D case is a straightforward extension) can be described in terms of a filter bank as shown in Fig. 1. After K levels of decomposition, reference signal $r_K(n)$ with resolution reduced by factor 2^K with respect to the original signal, as well as the detail signals $w_K(n)$, $w_{K-1}(n)$, ... $w_1(n)$ are obtained. Each detail signal $w_i(n)$ contains precisely the information that, together with the reference signal $r_i(n)$, enables reconstruction of $r_{i-1}(n)$, which is the reference signal at the next higher resolution. Figure 2 shows the tree levels wavelet decomposition of Lena image. The upper left block represents a smoothed approximation of the original image; its comes from three iterations of the low pass with down sampling. The other sub bands contain detail at various resolutions. Transform-based image compression is one of most successful applications of the wavelet transform. The wavelet representation is well matched to psycho visual models, and compression systems based on wavelet transform yield superior to other methods for given compression ratio.

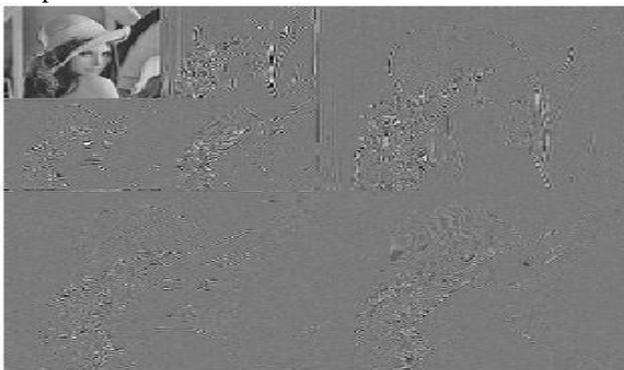


Figure 2: Wavelet Decomposition of the Image Lena

D. Water Marking in wavelet Domain

Watermarking in the wavelet domain is composed of two parts: encoding and decoding. In the encoding part, the original image and the watermark are first decomposed using wavelet pyramid structure. Then, the wavelet coefficients W , of the low-resolution representation of the watermark W , are embedded in the largest wavelet coefficients I_W of the low-resolution representation of the original image I , in the following way:

$$I^1_W = I_W(1 + \alpha W_W)$$

Spectrum analysis of the images reveals that most of the information in image is located in this low-resolution representation, which represents the smooth parts of the image. It is also known that human eyes are very sensitive to small changes in smooth part of the image. However, with the appropriate choice of the scaling parameter α , the invisibility of the watermark could be adjusted. Conversely, in case of possible attacks, the low-resolution representation of the watermark will still be preserved within the low-resolution representation of the image, which makes the watermark robust. Other coefficients of the watermark are embedded in the higher frequency components of the image, which represent the edges and textures of the image. Using above equation either will produce watermarked image that is not robust to image operations that perform low pass filtering (for small values of α) or will create visible defects in the images (for larger values of α). So in order to increase the robustness of the watermark, following equation is used:

$$I^1_W = I_W + \beta W_W$$

Since human eyes are not sensitive to small change in the edges and the textures of the image, invisibility of the watermark is kept. The watermarked image is obtained by applying inverse wavelet transform to the coefficients ' W^1 '. The watermarked image may then be subject to any number of distortions due to intentional or unintentional image processing operations. In the decoding process DWT of the suspected image. I and of the original (unwatermarked) image is performed. Wavelet coefficients of the low-resolution representation of the extracted watermark are obtained as:

$$W^-_{w=\alpha} = \frac{1}{\alpha} \left[\frac{I^-_w}{I_w} - 1 \right]$$

and wavelet coefficients in other frequency sub ands as:

$$W^-_{w=\beta} = \frac{1}{\beta} [I^-_w - I_w]$$

With inverse wavelet transform of W^- the extracted watermark W is obtained. Since, we use visually recognizable pattern as watermark, extracted watermarks can be compared with original watermark subjectively. Beside subjectively judgment for the watermark fidelity, we have defined an objective measure of similarity between the original watermark and the extracted watermark in the following way:

$$SIM = \frac{\sum_i 1 \sum_j w_{i,j} (W^-)_{i,j}}{\sum_i 1 \sum_j (w^-)_{i,j}^2}$$

Also, these measures could be applied in wavelet domain to each frequency sub band separately. This can benefit in the process of detection. For instance, applying any image

processing operation to the watermarked image that performs low pass filtering (compression, resizing), will result in lots of wavelet coefficients in higher frequency bands of the watermark. In this case, wavelet coefficients in lower frequency sub bands could be used to determine whether suspected image contains watermarks.

IV. CDMA

CDMA is a digital wireless air interface and networking standard based on the principle of spread-spectrum techniques, which allow multiple users to access the system simultaneously on the same carrier frequency. CDMA is a method in which users occupy the same time and frequency allocations, and are channelized by unique assigned codes. The signals are separated at the receiver by using a correlation that accepts only signal energy from the desired channel. Undesired signals contribute only to noise. In spread spectrum, the signal occupies a bandwidth in excess of the minimum necessary to send the information. The band spread is accomplished by means of a code that is independent of the data, and a synchronized reception with the code at the receiver is used for de-spreading and subsequent data recovery. Using Spread Spectrum, Processing gain = W/R can be achieved, where W is the final bandwidth of the spread message and R is the band-width of the baseband message. Spread-spectrum watermarking good performance stems from the relative immunity of the underlying spread-spectrum modulation to noise and jamming. Many improvements have been made to spread spectrum watermarking, both in embedding and in watermark detection and reading. One important development has been the application of perceptual models such as human visual system models, to the watermark embedding process. These perceptual models allow watermark signals to be added to a source with maximum efficiency. Another area of improvement has been in the development of filters for use prior to watermark detection and reading. In image watermarking, such filters are designed to suppress the host image and there by boost the watermark to host signal ratio. Equivalently, this amounts to estimating the host image and subtracting it from the watermarked image.

A. Spreading codes

Multiplication with the code sequence which is of a higher bit rate, results in a much wider spectrum. The ratio of the code rate to the information bit rate is called both the spreading factor and the processing gain of the CDMA system. The chipping rate is 1.2288 and the spreading factor is 64. Processing gain is usually given in dBs. To distinguish the information bit rate from the code rate, we call the code rate, chipping rate. In effect, it takes each data bit and convert it into k chips, which is the code sequence. It is called the chipping rate because the code sequence applied to each bit is as one can imagine it chipping the original bit into many smaller bits.

B. Spread Spectrum Method in Digital Water Marking

Another approach for watermarking utilizes the direct

sequence code division multiple access (DS-CDMA) spread spectrum communications. First, the watermark is transformed into a bit string $b_1b_2:::b_{64}$. In our case, you will need to transform your two dimensional watermark into a vector of length 64. For each bit b_i , a pseudorandom matrix R_i of integers $f_{i1}; 1g$ is generated (help rand). Before the generation, MATLAB's random number generator must be initialized using a predefined seed: $\text{rand('seed', seed)}$. This initialization enables the creation of exactly the same random matrices later on in the extraction phase. As a result, it will have 64 different matrices consisting pseudo randomly of ones and minus ones, each of the same size as the original image. Next, each of the R_i matrices will depend on the bit values b_i of the original watermark in the following way: A matrix $+R_i$ is used if b_i represents a 0, and a matrix R_i is used if b_i represent a 1. In other words, multiply the necessary R_i matrices. Thereafter, the sum of all random patterns R_i defines the watermark W :

$$\sum_{i=1}^{64} \pm R_i$$

Where the sign of each pseudo random matrix is dependent on the bit value as defined in the previous paragraph. Finally, the watermarked image IW is generated by adding the watermark into cover image I .

$$Iw = I + kW$$

Where the scalar k is a gain factor which can be used to control the watermark strength. The embedded watermark can be extracted one bit at a time by calculating the correlation between normalized watermarked image and the pseudorandom pattern R_i . In this exercise we will calculate the correlation C_i between the normalized image IOW and the pseudorandom pattern R_i as

$$C_i = \sum((R_i - R_i^-)Iw)$$

Where R_i is average of all values in R_i . The operator ϕ is used for element-wise matrix multiplication, and the operator \sum to calculate a sum of matrix elements. The normalized watermarked image is defined as,

$$Iw = (Iw - Iw^-)$$

Where Iw is the average of all values in Iw . If the resulting correlation C_i is positive the value 0 is assigned to the corresponding bit, and otherwise the value 1 is used. Making function embed CDMA for embedding a watermark according to the spread spectrum method. The function should take four inputs: a cover image, a watermark, parameter k (the gain factor), and seed for the random number generator. Thereafter, make function extract CDMA for extracting the watermark. The function should take as input a watermarked image and a seed for random number generator. In order to get the same random numbers in embedding and extraction functions, one would need to give the same seed as an input parameter for both functions. Embed your watermark with CDMA method into an image using value 1 for the gain factor k . Thereafter, add Gaussian noise with zero mean into CDMA watermarked image and increase the noise variance gradually. Make sure that the variable type of the image (double or uint8) is correct before adding the noise to avoid problems. Thereafter, compress the original watermarked image with lossy JPEG. Raise the compression rate (reduce the quality) gradually. High compression rate one can use so that one can still extract undamaged watermark.

V. CONCLUSIONS

This study has introduced a number of techniques for the watermarking of digital images. We have seen the three techniques of DCT, DWT & CDMA. Embedding in the DCT domain proved to be highly resistant to JPEG compression as well as significant amounts of random noise. By anticipating which coefficients would be modified by the subsequent transform and quantization, we were able to produce a watermarking technique with moderate robustness, good capacity, and low visual impact. In DWT whole image comparison takes place so furthermore, this greatly reduces our flexibility, as promising domains such as the DWT. In CDMA the sequence is in binary form so due to introduction of pseudo noise during embedding and recovery it takes more time. As per the comparison of time analysis graph CDMA is more secure than DCT, DWT. As security increases time to embed and recover also increases.

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